Physics II
ISI B.Math
Final : April 25, 2003

Answer question 1 and any five out of the remaining. Each question carries 10 marks.
Total marks : 60

1. Answer whether each of the following statements are TRUE or FALSE and explain your answer briefly. A few sentences should be sufficient.

a) For an ideal monatomic gas, the pressure is given by $p = 2/3u$ where u is the energy density.
b) Given two equilibrium states, one can *always* find an irreversible path connecting the two states for which the entropy change is *larger* than the entropy change along a reversible path connecting the same two states.
c) In any adiabatic process the entropy of a system does not change.
d) A classical ideal gas violates the third law of thermodynamics.
e) A Carnot engine whose thermal efficiency is very high is particularly well suited for a refrigerator, if run in the reverse direction.
f) The absolute temperature of an isolated system can never be negative.
g)Two isolated systems are characterized by different chemical potentials μ_1 and μ_2. The systems are then placed in diffusive contact. To reach thermodynamic equilibrium, particles will diffuse from the region of higher chemical potential to the region of lower chemical potential.
h)Liquid water and water vapour are in equilibrium in a cylinder at 100°C and at 1 atm pressure. If the piston is pushed down slowly on the system, the pressure of the liquid-vapour mixture will increase.
i)If a system is in equilibrium with a heat reservoir at constant temperature its specific heat is necessarily positive.
j)The expression $S = Nk(lnV + \frac{3}{2}lnT + C)$ for the entropy of an ideal gas (where C is a constant independent of T, V, N) is consistent with the fact that entropy is an extensive quantity.

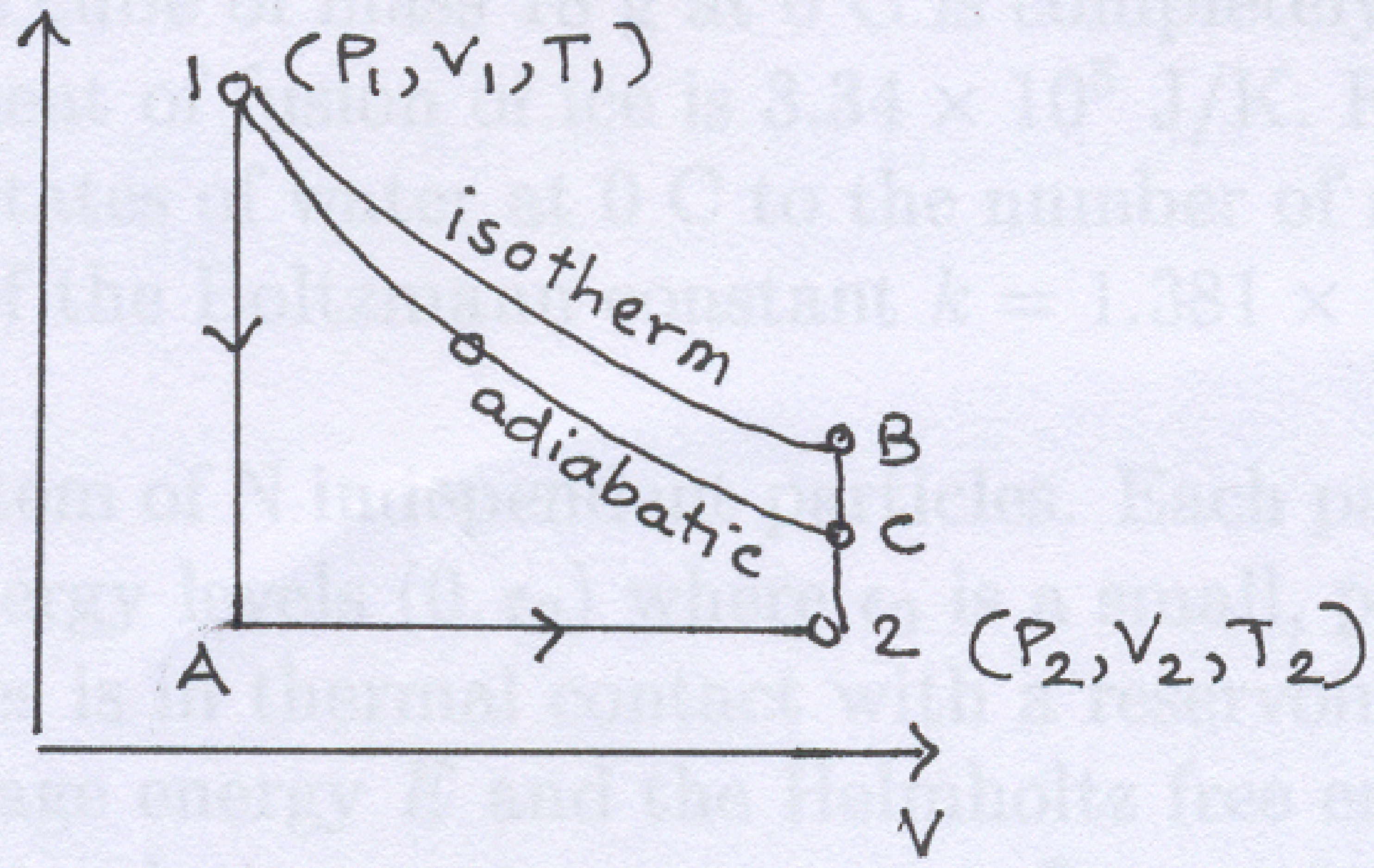
2. A room air conditioner operates as a Carnot cycle refrigerator between an outside temperature T_h and a room at alower temperature T_l. The room gains heat from outdoors at a rate $A(T_h - T_l)$; this heat is removed by the

air conditioner. The power supplied to the cooling unit is P.

(a) Show that the steady state temperature of the room is

$$T_l = (T_h + P/2A) - [(T_h + P/2A)^2 - T_h^2]^{\frac{1}{2}}$$

(b) If the outdoors is at 37°C and the room is maintained at 17°C by a cooling power of 2kW, find the heat loss coefficient A of the room in W/K.



3. A fixed mass (n moles) of ideal gas is changed from a state (P_1, V_1, T_1) to a state (P_2, V_2, T_2) by the following three quasistatic processes as seen in the figure: (i)1A2 (ii) 1B2 and (iii) 1DC2. What is the increase in internal energy for $1 \to 2$? Also obtain the work that must be done on the system and the heat that must be added in each process. Assume that the heat capacity at constant volume C_V is constant and $\frac{C_P}{C_V} = \gamma$. Express your results in terms of $(P_1, V_1, T_1), (P_2, V_2, T_2), C_V$ and γ

4. (a) For a PVT system show that

$$\left(\frac{\partial U}{\partial V}\right)_P = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

[Hint: Use the fact that dS is an exact differential]

(b) The equation of state for a van der Waals gas is given by

$$P = \frac{RT}{V - b} - \frac{a}{V^2}$$

where a and b are constants. Assuming that the heat capacity at constant volume C_V is a constant, use the result in part (a) to derive the internal energy and entropy for a van der Waals gas.

(c) Show that for an adiabatic transformation of a van der Waals gas

$$T(V-b)^{\frac{R}{C_V}} = constant$$

5.a) N particles are distributed among 3 states having energies $E = 0, E = kT$ and $E = 2kT$. If the equilibrium energy of the system is $1000kT$, what is the value of N ?

b) Suppose an ice cube of mass 18 g at 0 C is completely melted to water at 0 C. The latent heat of fusion of ice is 3.34×10^5 J/K. Find the ratio of the number of microstates of water at 0 C to the number of microstates of ice at 0 C. [The value of the Boltzmann constant $k = 1.381 \times 10^{-23}$ J/K]

6. Consider a system of N independent particles. Each particle can have only one of the two energy levels $(0, \epsilon_0)$ where ϵ_0 is a small, positive energy. This system of particles is in thermal contact with a reservoir of temperature T.

(a) Find the average energy $\bar{E}$ and the Helmholtz free energy of the system.

(b) Show that the relative root mean square fluctuation of energy for this system $\frac{\Delta E}{\bar{E}}$ is negligible when N is large. $((\Delta E)^2 = \overline{(E - \bar{E})^2}$)

7. For electromagnetic radiation in thermal equilibrium at temperature T in a metallic cavity the energy of a mode of angular frequency ω is given by $\epsilon_n = n\hbar\omega$ where n is the number of photons in the mode $(n = 0, 1, 2, \cdots)$.

(a) Find the average number of photons in a single mode of frequency ω.

(b) Find the average energy in the above mode. What is the classical limit of this average energy ?

(c) Given that the spectral density of this radiation, i.e, the energy density per unit angular frequency range is given by

$$u_\omega = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{\exp\frac{\hbar\omega}{kT} - 1}$$

show that the entropy density of photons at the temperature T is proportional to T^3. [You may need to use $\int_0^\infty dx \frac{x^3}{e^x - 1} = \frac{\pi^4}{15}$]